Security of Keyed Sponge Constructions Using a Modular Proof Approach

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Fast Software Encryption
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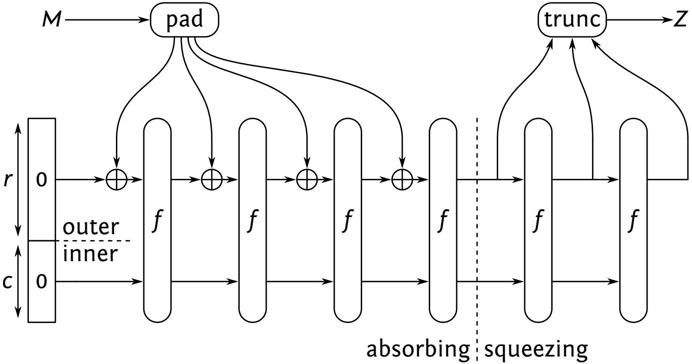
Sponges



- Hashing
- Keyed applications

Keyed Sponges

Stream cipher encryption

- Squeezing $k = \text{Sponge}(K||\text{nonce})$
- Block-wise $k_i = \text{Sponge}(K||\text{nonce}||i)$

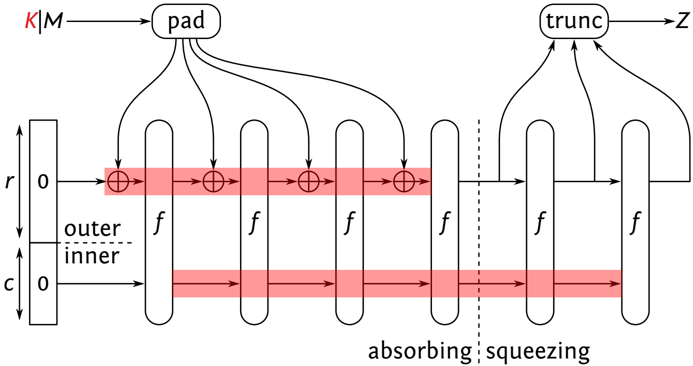
Authentication

- $\text{MAC} = \text{Sponge}(K||M)$

Authenticated encryption

- Duplexing the sponge

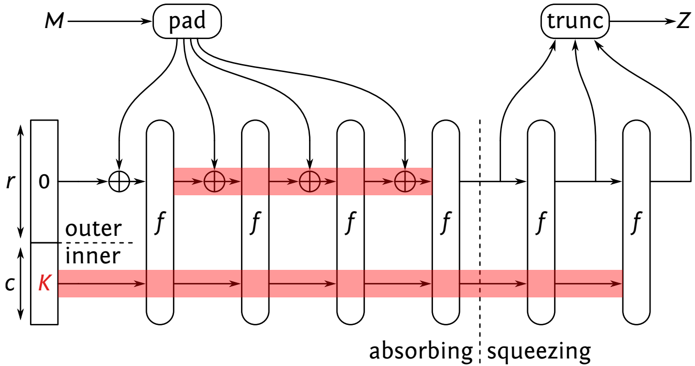
Outer-Keyed Sponge [BertoniDPV11]



$$\mathsf{OKS}^f_K(M) = \mathsf{Sponge}^f(K||M)$$

(K of length a multiple of r)

Inner-Keyed Sponge [ChangDHKN12]



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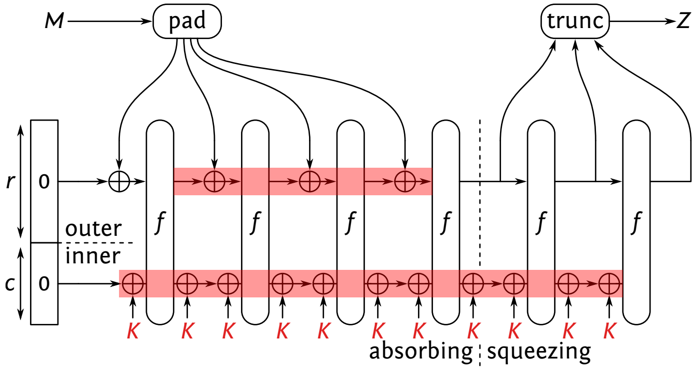

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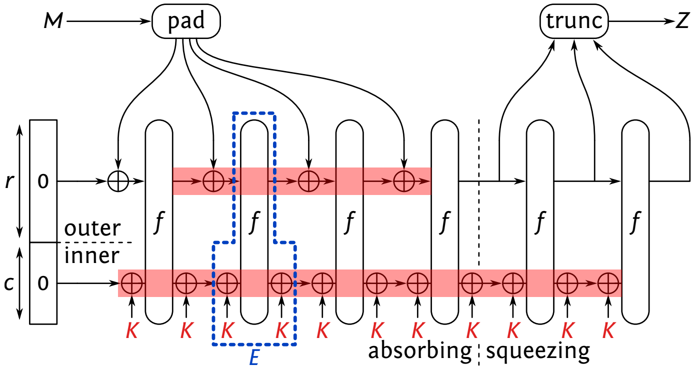

$$\mathsf{IKS}_K^f(M) = \mathsf{Sponge}^{E_K^f}(M)$$

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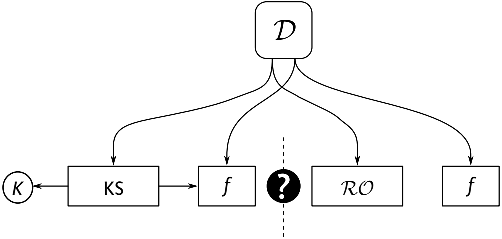
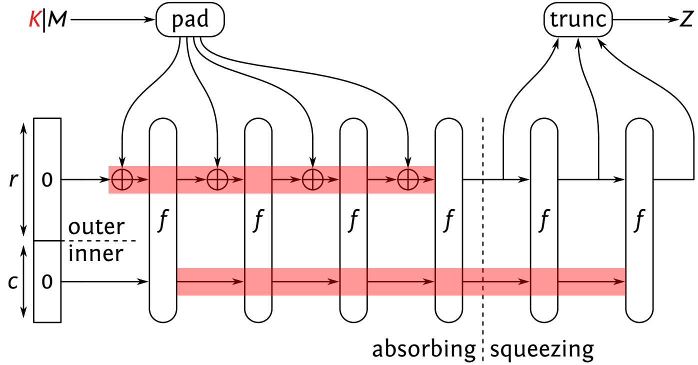
Security Model



- M: online **data** complexity (blocks)
  - Calls to KS_K or $\mathcal{RO}$
- N: offline **time** complexity
  - Calls to f

Existing Distinguishing Bound [BertoniDPV11]



$$\mathbf{Adv}_{\mathsf{OKS}} \leq \frac{M^2}{2^{c+1}} + \frac{2MN}{2^c} + \frac{N}{2^k}$$

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Bad news

- Flaw in Lemma 1 of [BertoniDPV11]
- Easily fixable, but adds additional term

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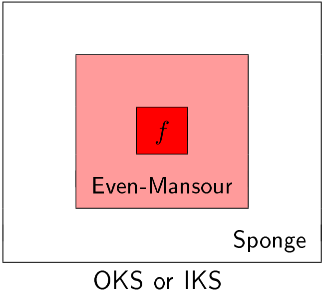
Good news

- Different proof approach leads to better results

Modular Proof Approach

Proofs based on reduction to underlying primitives



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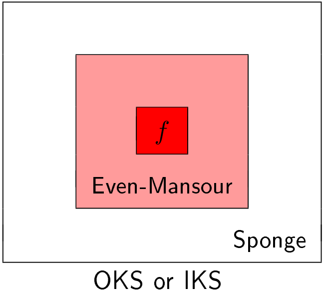
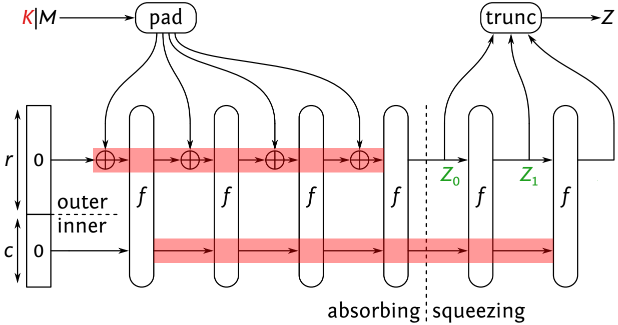


- Easier proofs
- Better bounds
- More general due to use of multiplicity

Multiplicity [BertoniDPV10]



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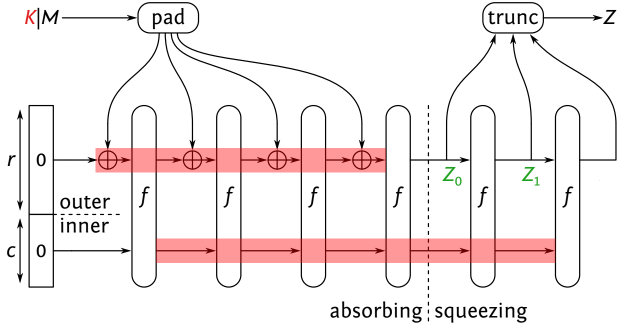

$$\mu_{\text{fw}} : \max_{Z_0} \text{ \# evaluations } f(Z_0||?) = (?||?)$$

$$\mu_{\text{bw}} : \max_{Z_1} \text{ \# evaluations } f(?||?) = (Z_1||?)$$

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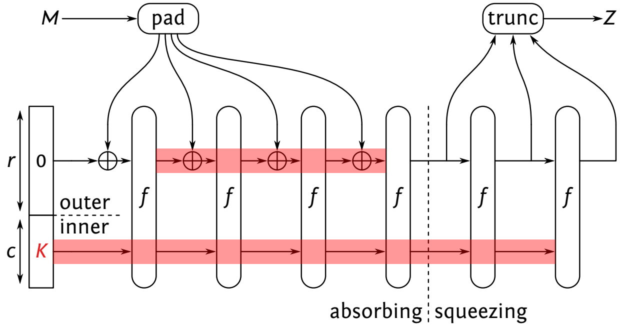
- $M/2^r \leq \mu_{\text{fw}}, \mu_{\text{bw}} \leq M$
  - General case: close to M
  - Constrained case (unique nonce): close to $M/2^r$

Inner-Keyed Sponge



$$\mathbf{Adv}_{\mathsf{IKS}} = \Delta(\mathsf{IKS}_K^f, f; \mathcal{RO}, f)$$

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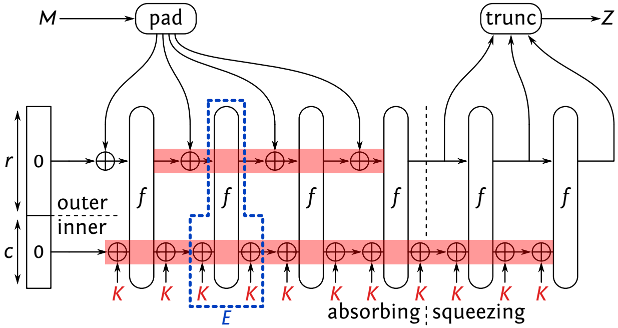

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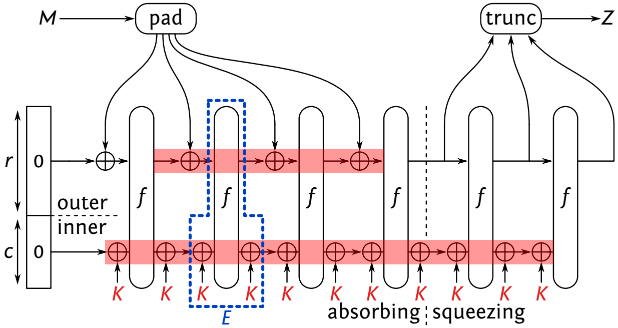

$$
\begin{aligned}
\mathbf{Adv}_{\mathsf{IKS}} &= \Delta(\mathsf{IKS}^{f}_{K}, f; \mathcal{RO}, f) \\
&= \Delta(\mathsf{Sponge}^{E^{f}_{K}}, f; \mathcal{RO}, f) \\
&\leq \Delta(\mathsf{Sponge}^{\pi}, f; \mathcal{RO}, f) + \Delta(E^{f}_{K}, f; \pi, f)
\end{aligned}
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- Independent of f
- Indifferentiability bound of sponge

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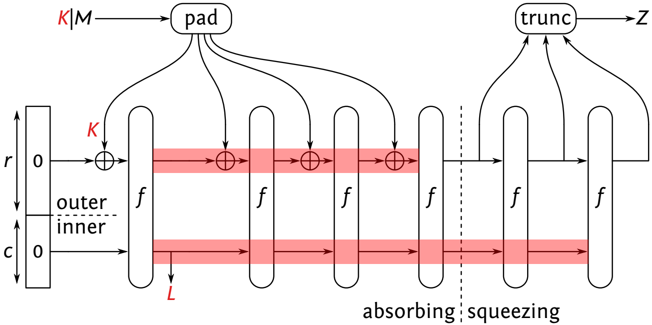

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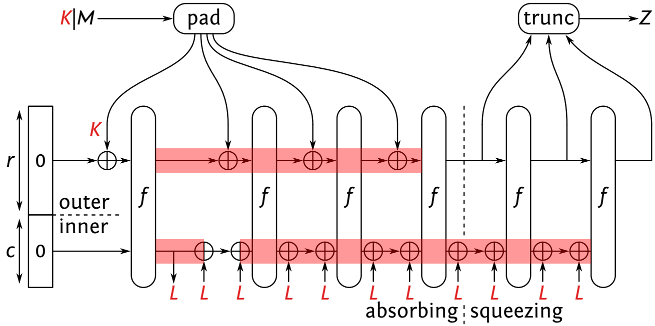

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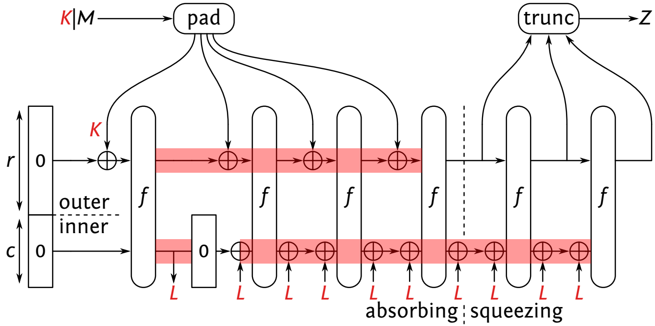

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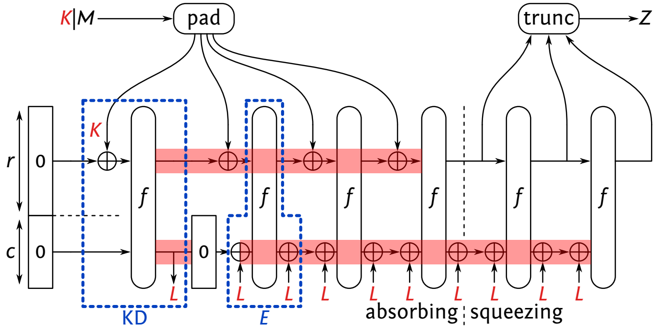

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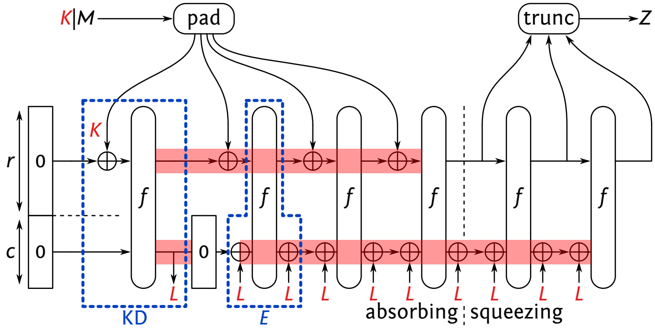

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- PRP-security of Even-Mansour with c-bit subkey
- Analysis more technical
  - If all calls to f in $\mathsf{KD}^f(K)$ unique (term 1)
  - Probability an f-call in $\mathsf{KD}^f(K)$ collides (rest)

Interpretation

- Dominating term:

$$\mathbf{Adv}(M, \mu_{\mathrm{fw}}, \mu_{\mathrm{bw}}, N) \leq \frac{M^2}{2^c} + \frac{2(\mu_{\mathrm{fw}} + \mu_{\mathrm{bw}})N}{2^c} + \frac{N}{2^k}$$

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$$\Downarrow$$

Time complexity is $\min\{2^{c-\beta-1}, 2^k\}$

Interpretation

$$\frac{2(\mu_{\mathrm{fw}} + \mu_{\mathrm{bw}})N}{2^c} \leq \frac{2(\mu_{\mathrm{fw}}^* + \mu_{\mathrm{bw}}^*)N}{2^c} + \Pr\left(\mu_{\mathrm{fw}} > \mu_{\mathrm{fw}}^*\right) + \Pr\left(\mu_{\mathrm{bw}} > \mu_{\mathrm{bw}}^*\right)$$

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General case

- μ_{fw} may be up to M (adversary has full control)
- μ_{bw} at most $\mathrm{const} \cdot \max\{1, M/2^r\}$
  - except with small probability

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Interpretation (Ignoring 2^k Term)

rate	capacity	data complexity	case	time complexity
r	c	$\leq 2^{\alpha}$	general	$2^{c-\alpha-1}$
			constrained	$\min\{2^{c-2}, 2^{r+c-\alpha-2}\}$
40	160	$\leq 2^{79}$	general	2^{80}
			constrained	2^{119}
548	252	$\leq 2^{123}$	general	2^{128}
			constrained	2^{250}

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- System with n independent keys
- Damage if any instance is broken

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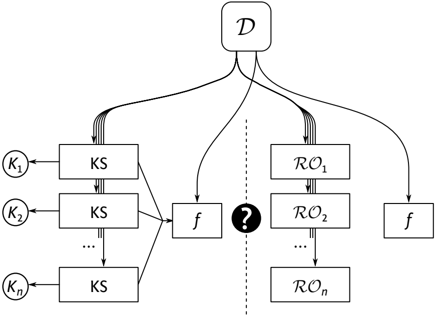

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- Exhaustive key search speed-up

$$\frac{N}{2^k} \quad \longrightarrow \quad \frac{nN}{2^k}$$

Conclusion

Thanks for your attention!

Questions?