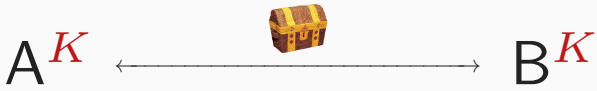
Challenges in Symmetric Cryptography

Bart Mennink
Radboud University (The Netherlands)
Finse Cybersecurity Winter School 2025
April 28, 2025

Keyed Symmetric Cryptography

General Setting



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  - They have agreed on a joint key K and use it to transmit data

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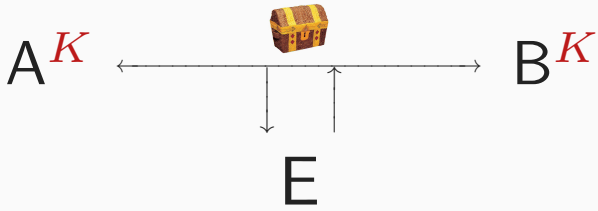

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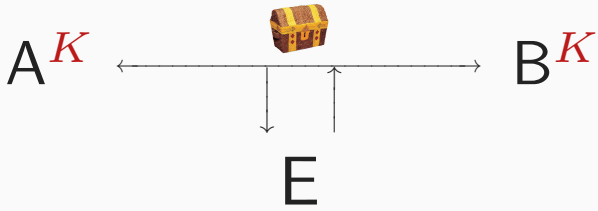

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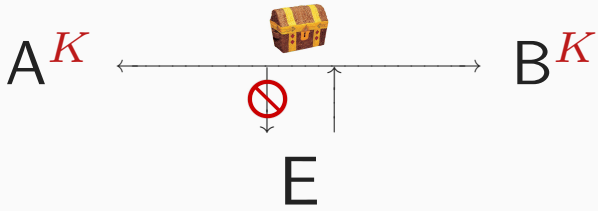

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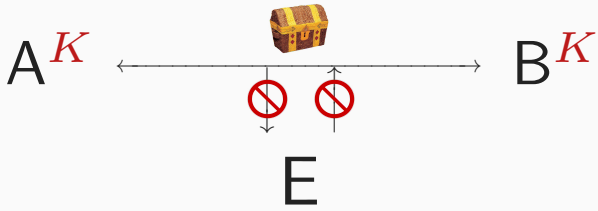

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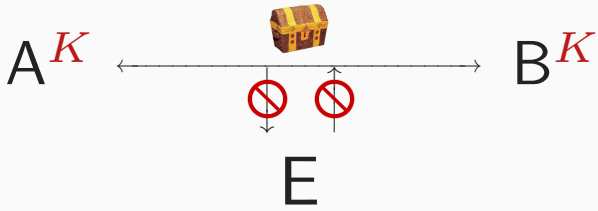

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Authenticated encryption aims to achieve both

One-Time Pad Encryption

Encryption:

$$
\begin{array}{lcccccccccccccccc}
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\end{array}
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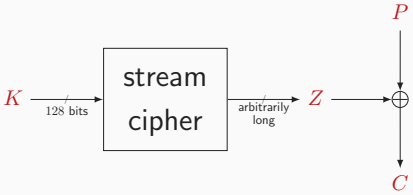
Stream Ciphers

- Generate long keystream Z from short key K



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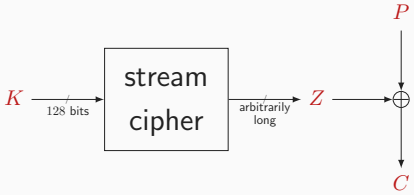

- Security degrades:
  1. Key guessing still succeeds with probability $1/2^{|K|}$ but now with shorter key
  2. The stream cipher mechanism is another focal point of attack

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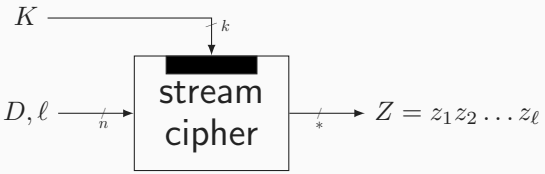
We need something more sophisticated!

Modern Stream Cipher Design

Modern Stream Ciphers



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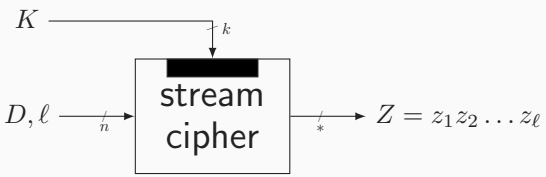

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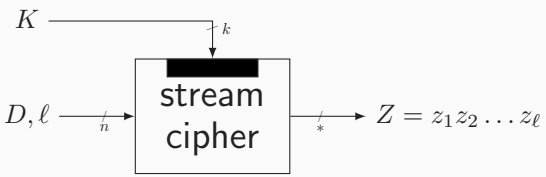

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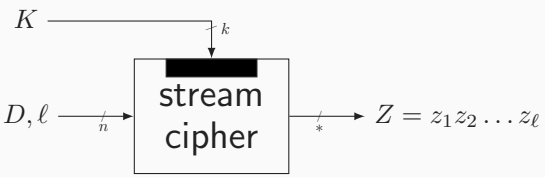
- Example: data streams, e.g., pay TV and telephone, often split data in relatively short, numbered, frames. The frame number may serve as diversifier:

$$C_i = P_i \oplus \mathsf{SC}(K, i, |P_i|)$$



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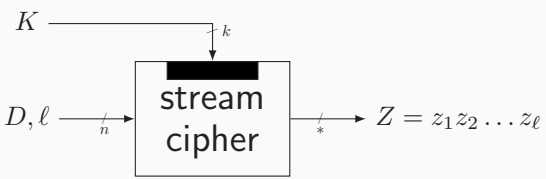
When is a stream cipher strong enough?

Stream Cipher Security, Intuition (1/3)



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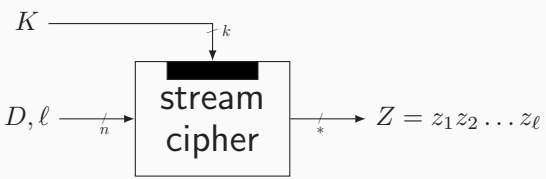

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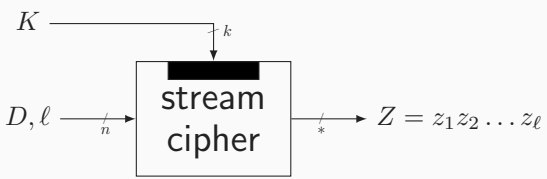

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- SC_K should behave like a random oracle

Intermezzo: Random Oracle

Random Oracle

- A database of input-output tuples
- Initially empty

D	Z
...	...
...	...
...	...
...	...

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    - update (D, Z) in the list

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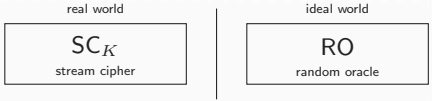
Stream Cipher Security, Intuition (2/3)



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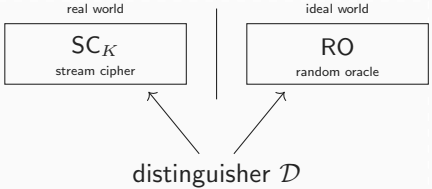

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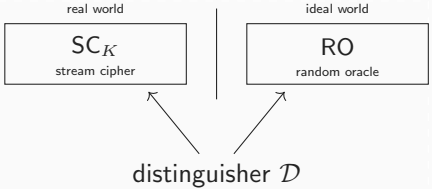

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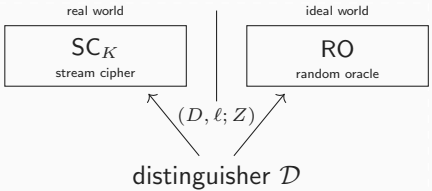

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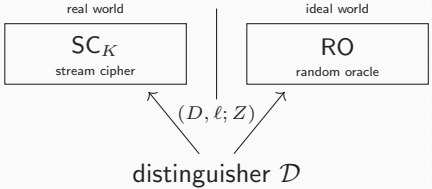

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  - At the end, $\mathcal{D}$ has to guess the outcome of the toss coin (head/tail)

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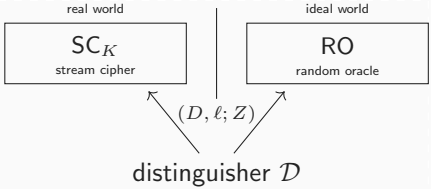

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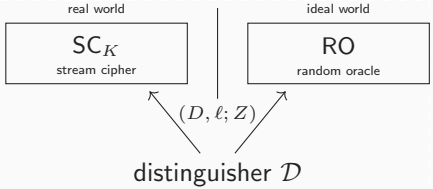

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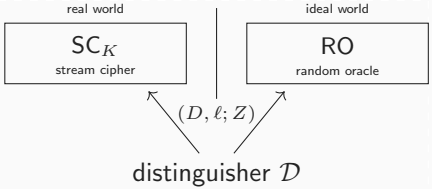

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  $$\mathbf{Adv}(\mathcal{D}) = 2 \cdot \mathbf{Pr}\,(\text{success}) - 1 = \mathbf{Pr}\left(\mathcal{D}^{\mathsf{SC}_K} \text{ returns head}\right) - \mathbf{Pr}\left(\mathcal{D}^{\mathsf{RO}} \text{ returns head}\right)$$

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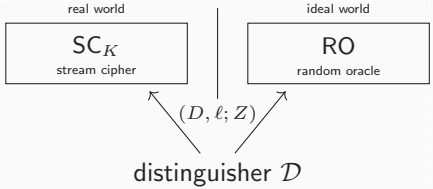

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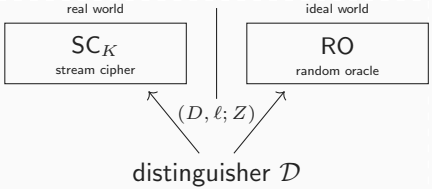
- $\mathcal{D}$ can always guess and succeeds with probability $\geq 1/2$, so we scale the probability to $\mathcal{D}$'s advantage:

  $$\mathbf{Adv}(\mathcal{D}) = 2 \cdot \mathbf{Pr}\,(\text{success}) - 1 = \mathbf{Pr}\left(\mathcal{D}^{\mathsf{SC}_K} \text{ returns head}\right) - \mathbf{Pr}\left(\mathcal{D}^{\mathsf{RO}} \text{ returns head}\right)$$

- $\mathcal{D}$ is limited by certain constraints

Stream Cipher Security, Intuition (3/3)



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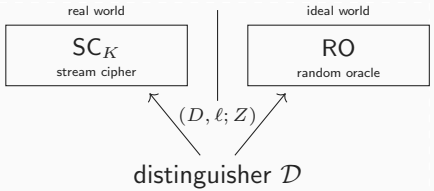

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Generic Stream Cipher Design

Generic Stream Cipher Design (1/2)

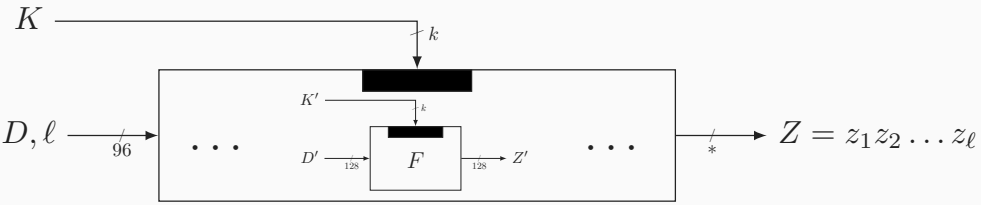
Modern Approach

- Stream cipher built from smaller cryptographic primitive

Generic Stream Cipher Design (1/2)

Modern Approach

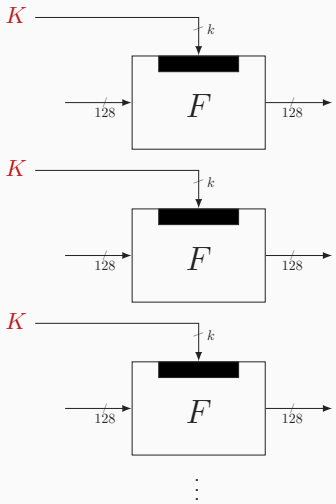
- Stream cipher built from smaller cryptographic primitive
- Suppose (for the sake of argument):
  - we **know** how to build a strong stream cipher F with fixed-length output
  - we **want** to build a stream cipher with variable-length output



Generic Stream Cipher Design (2/2)

Design

- Feed K to primitive



Generic Stream Cipher Design (2/2)

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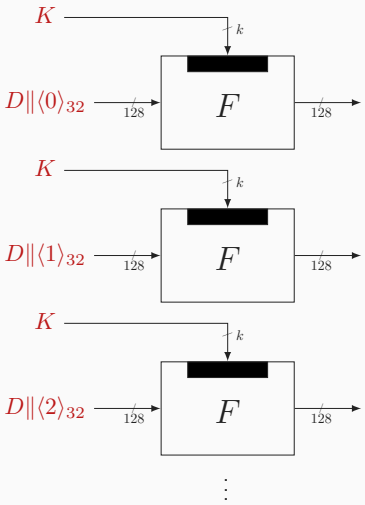

Generic Stream Cipher Design (2/2)

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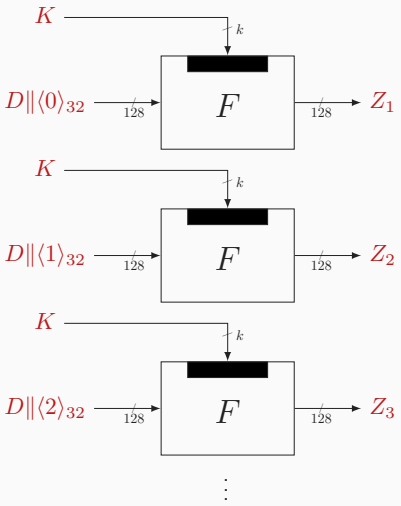
$$Z = Z_1 \parallel Z_2 \parallel Z_3 \parallel \cdots$$



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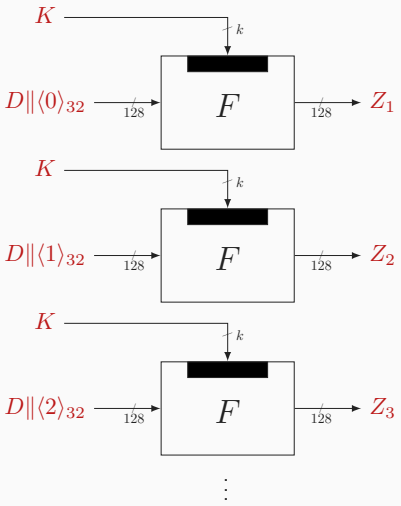

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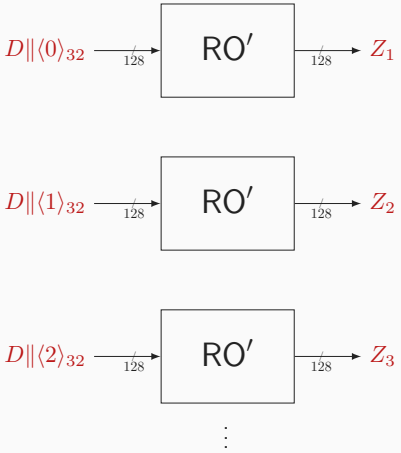

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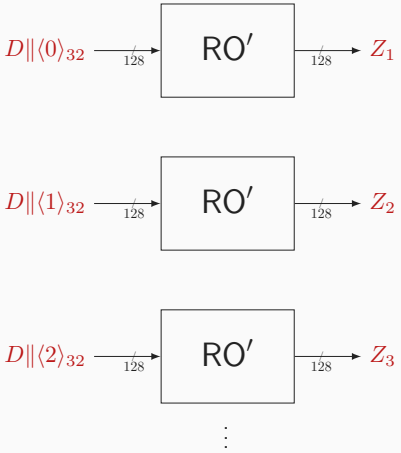

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- Then construction is hard to distinguish from RO

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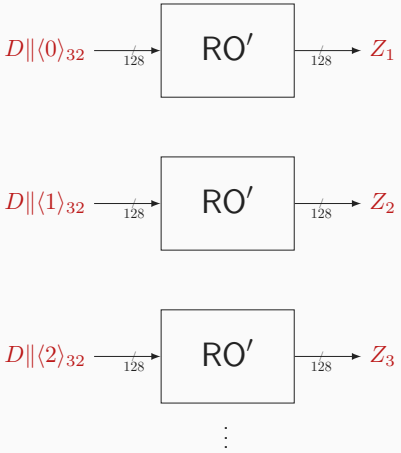

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- For the purists: $\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{SC}[F]}(q,t) \leq \mathbf{Adv}^{\mathrm{prf}}_{F}(q,t')$

Generic Stream Cipher Design (2/2)

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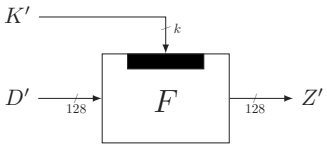
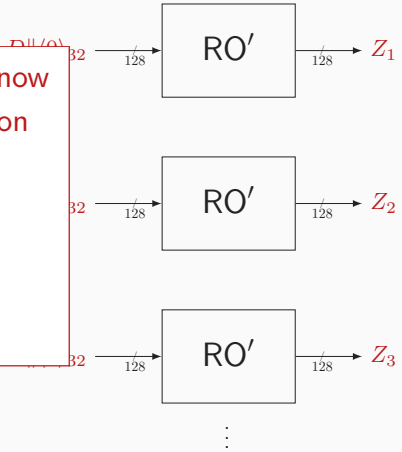
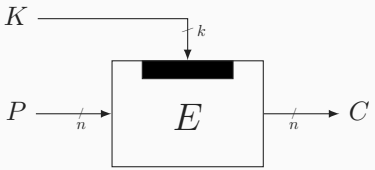


that behaves like a RO′



Block Ciphers



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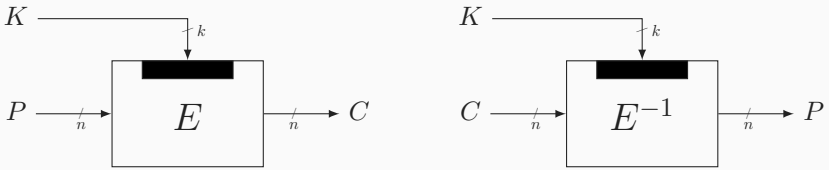

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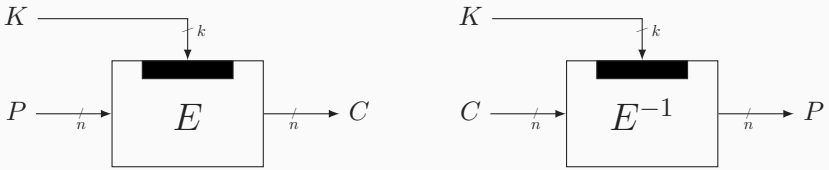

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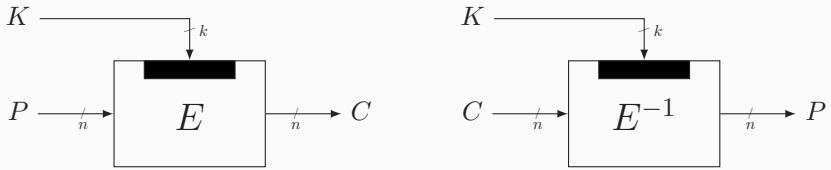

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- A good block cipher should behave like a random permutation
  - We denote the advantage of distinguisher $\mathcal{D}$ by $\mathbf{Adv}_E^{\mathrm{prp}}(\mathcal{D})$
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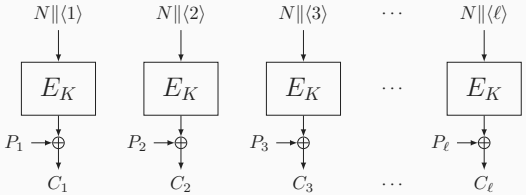
Counter (CTR) Mode



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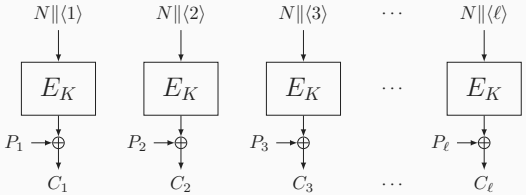

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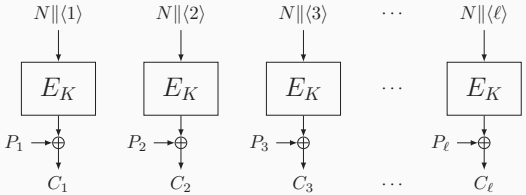

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birthday bound

General Birthday Paradox

- Randomly draw q elements from sample space $\{0,1\}^n$
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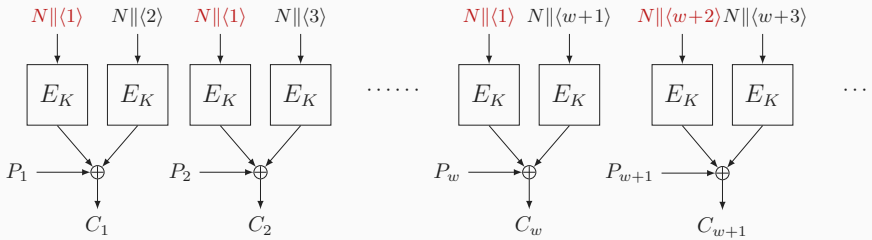
- Lightweight block ciphers have a block size of 64 bits or even less [BKL+07]

CENC [Iwa06]



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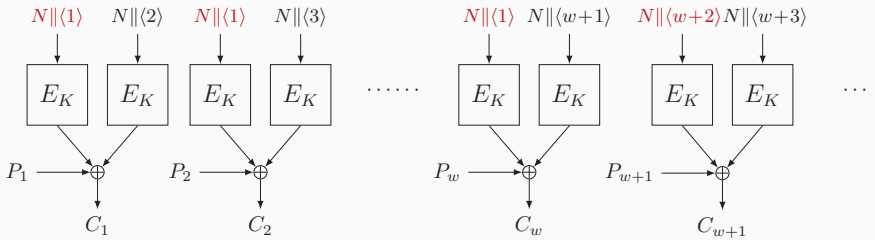

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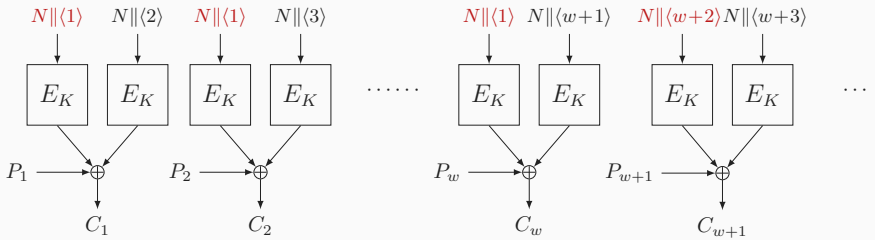

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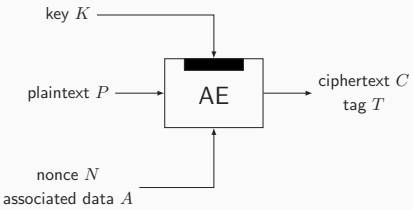
$$\mathbf{Adv}^{\mathrm{prf}}_{\mathsf{CENC}}(q,t) \leq \mathbf{Adv}^{\mathrm{prp}}_{E}((w+1)q,t') + wq/2^n$$

Authenticated Encryption and GCM

Authenticated Encryption



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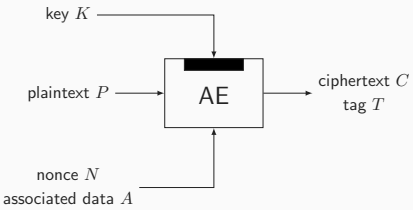

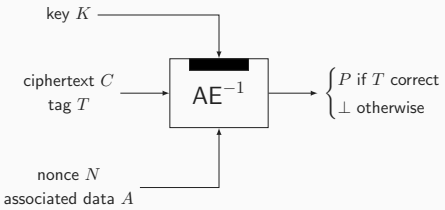

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- Unwrapping needs to satisfy that
  - Plaintext disclosed if tag is correct
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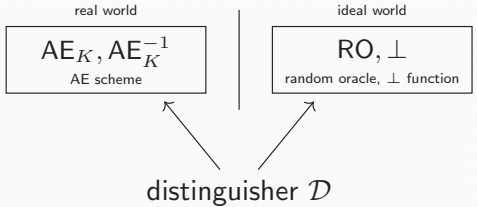
Authenticated Encryption Security



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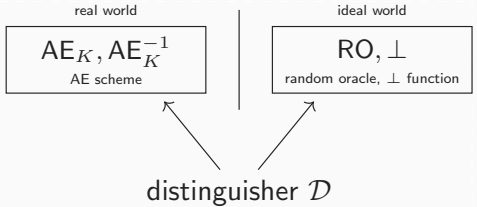

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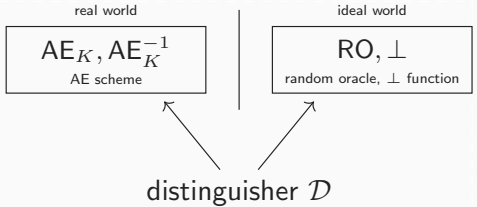

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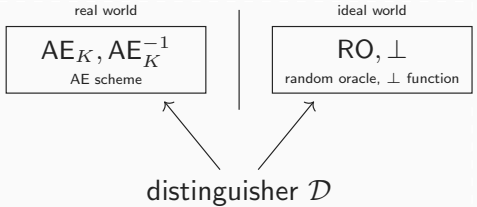

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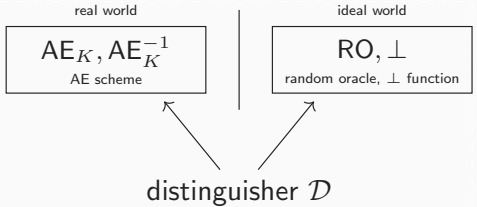

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Universal Hash Functions

- Consider a function $H : \{0,1\}^k \times \{0,1\}^* \to \{0,1\}^t$
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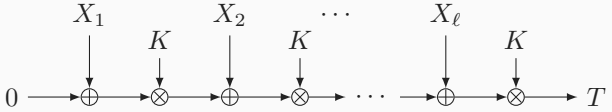
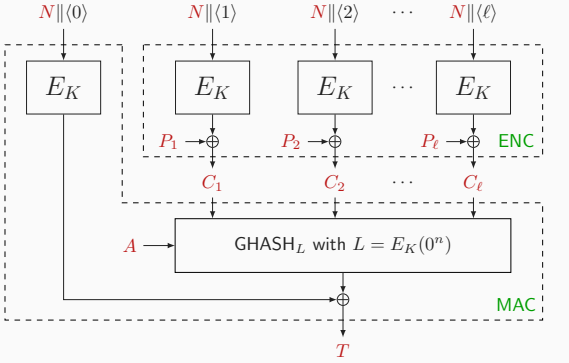
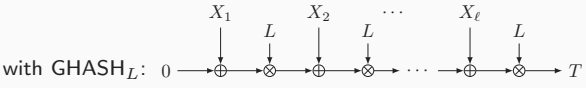
GHASH

- Addition and multiplication over finite field (with proper input encoding)
- $\ell 2^{-k}$-XOR-universal [MV04]



GCM for 96-bit Nonce N [MV04]





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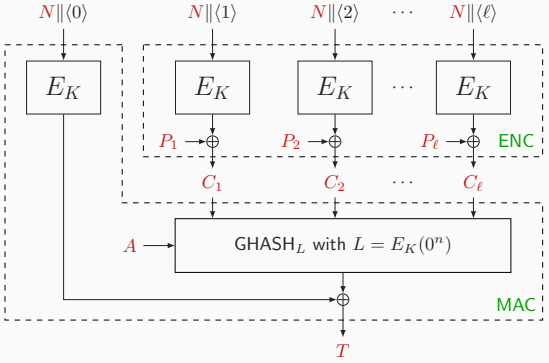

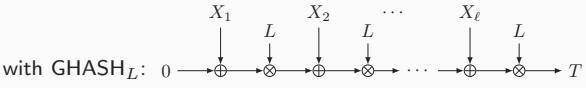

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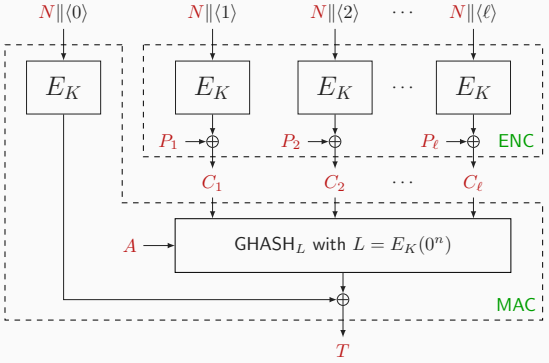

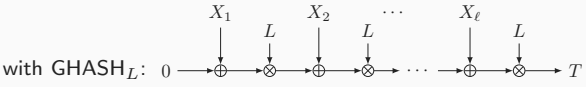

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- Equally popular: ChaCha20-Poly1305!

- Parallelizable
- Evaluates E only (no E^{-1})
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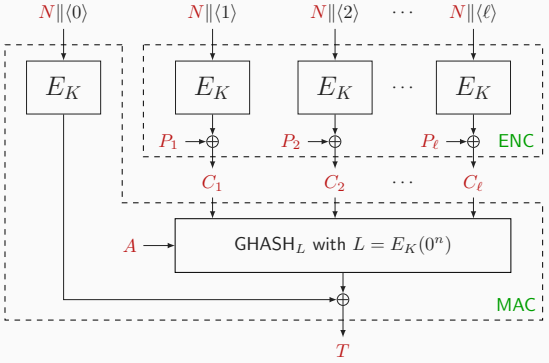
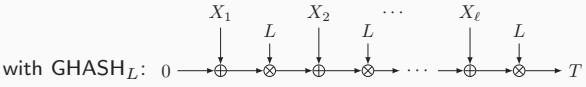
- Nonce reuse is devastating
  - Leaks $P \oplus P' = C \oplus C'$ and L

Problems With GCM for 96-bit Nonce N (1/2)





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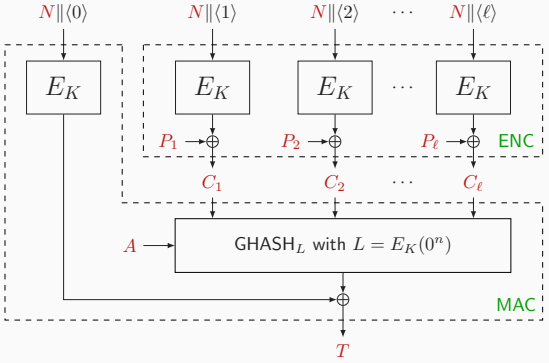

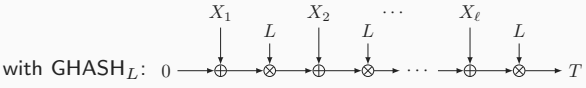

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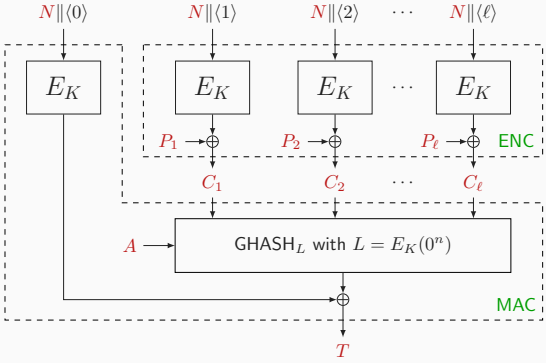

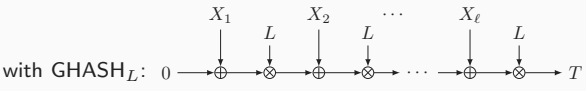

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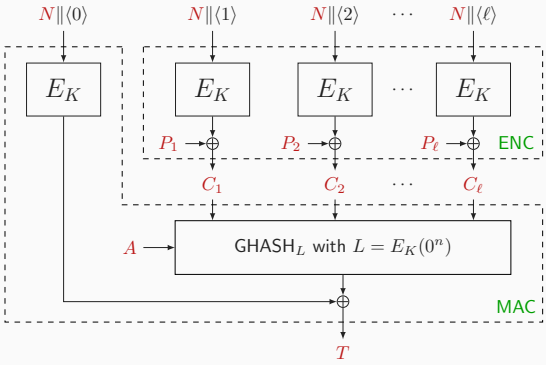

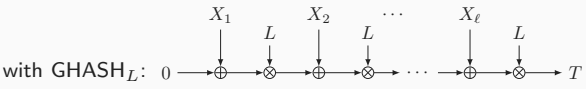

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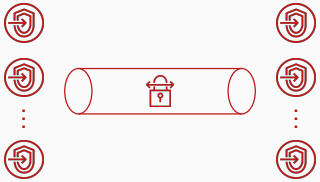
Short Nonce

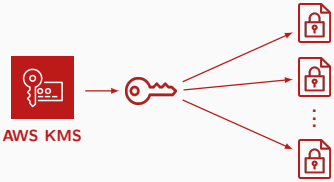
- Biggest problem of all

Problems With GCM for 96-bit Nonce N (2/2)

Use Case: Amazon Web Services [KCCP23]



AWS distributed systems encrypt
up to $\approx 2^{32}$ plaintexts per 2 seconds



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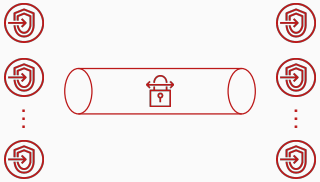

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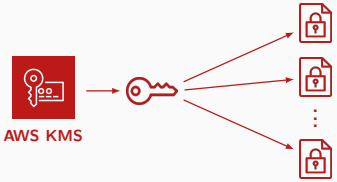

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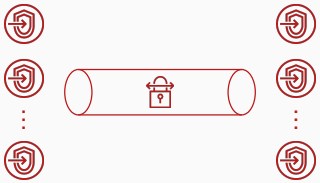

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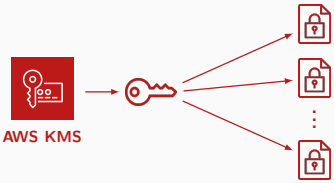

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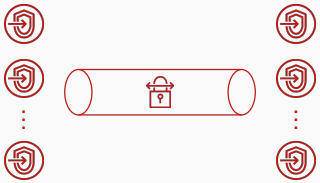

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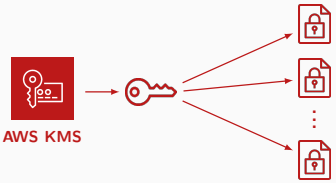

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- AWS also needs 32-bit identifiers, leaving only 64 bits of nonce

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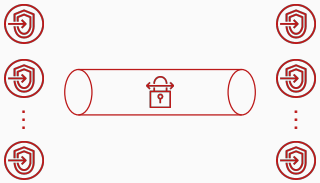

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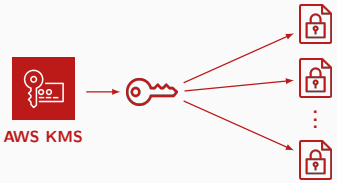

AWS KMS key encrypts
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- Counter management is problematic
- Random 96-bit nonces tend to collide
- AWS also needs 32-bit identifiers, leaving only 64 bits of nonce
- Limitations of GCM are a real world problem!

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Symmetric Cryptography

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- Certain use cases expose limitations of current standards

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Outline

- Accordion modes $\longrightarrow$ ddd-AES
- Lightweight cryptography $\longrightarrow$ Ascon

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