Anonymous Credential Schemes with Encrypted Attributes

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joint work with

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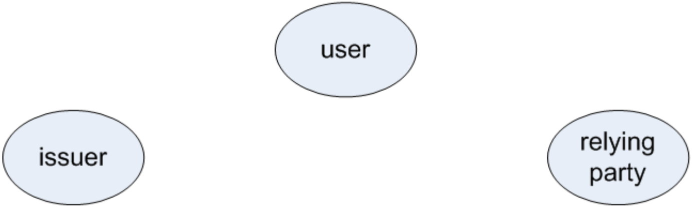
Outline

Anonymous Credential Schemes



- 3 types of parties: *issuers, users, relying parties (verifiers)*

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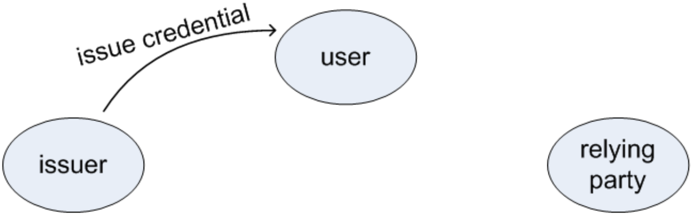

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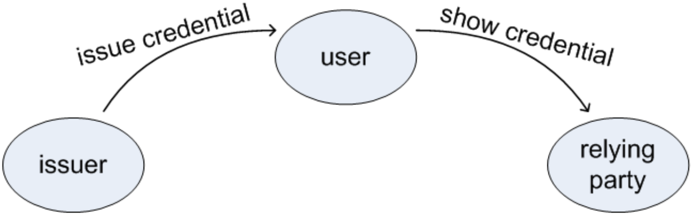

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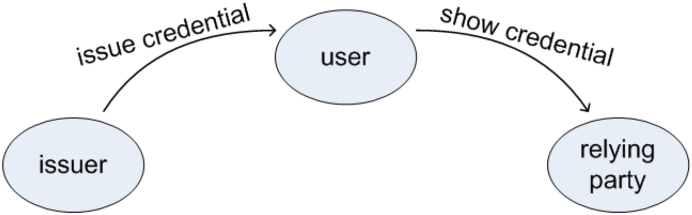


- 3 types of parties: *issuers, users, relying parties (verifiers)*
- Issuer issues credential (on some attributes) to user
- User shows credential to a relying party
- Required security properties: unforgeability and unlinkability
- Examples: digital passports, identity management, ...

Anonymous Credential Schemes

Theory:

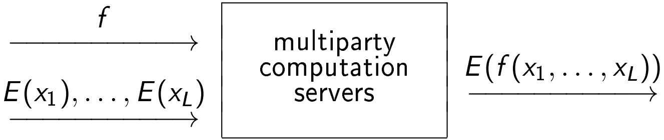
- Introduced by Chaum in 1982-1986
- Several efficient constructions, most prominently by
  - Brands [Bra93, Bra95, Bra99]
  - Camenisch-Lysyanskaya [CL01, CL02, CL04]
- Plus variations and extensions

Practice:

- eCash, DigiCash (Chaum)
- CAFE project
- Idemix (IBM, Camenisch-Lysyanskaya credentials)
- U-Prove (Microsoft, Brands credentials)

Secure Function Evaluation (SFE)

- A number of parties jointly and securely compute a function f on secret data



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$$\begin{array}{c} \xrightarrow{\quad f \quad} \\ \xrightarrow{E(x_1), \ldots, E(x_L)} \end{array} \boxed{\begin{array}{c}\text{multiparty}\\ \text{computation}\\ \text{servers}\end{array}} \xrightarrow{E(f(x_1, \ldots, x_L))}$$

Yao's millionaires protocol (1982)

- Alice and Bob want to compare their wealth

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- Both encrypt their fortunes: $E(x_A), E(x_B)$

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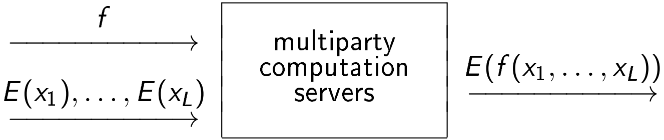

Yao's millionaires protocol (1982)

- Alice and Bob want to compare their wealth
- Both encrypt their fortunes: $E(x_A), E(x_B)$
- Jointly execute SFE protocol to compute $x_A < x_B$
- Security: the protocol should not leak any other info about x_A, x_B

Secure Function Evaluation (SFE)

Theory:

- Introduced by Yao in 1982-1986
- Main approaches to multiparty computation by
  - Ben-Or *et al.* [BGW87]
  - Goldreich *et al.* [GMW87]
  - Cramer *et al.* [CDN01]

Practice:

- E-voting, e-auctions, ...
- Fairplay (2004), VIFF (2007), Sharemind (2008), and more
- Commercial activity: new company Partisia

Combining Credential Schemes and SFE

Problem:

- Input to SFE should be correct or meaningful
  - → One can employ credentials to guarantee this
  - → But SFE servers are not allowed to learn the attributes, while standard credential schemes require the user to know these

Combining Credential Schemes and SFE

Problem:

- Input to SFE should be correct or meaningful
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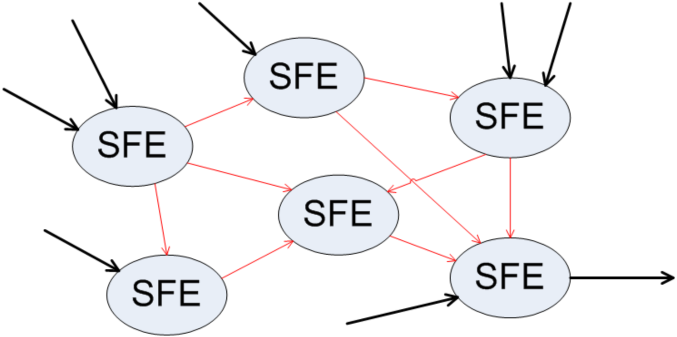
Solution:

Anonymous Credentials on Encrypted Attributes

Combining Credential Schemes and SFE

- Lead to networks of SFEs with anonymous links connecting inputs and outputs



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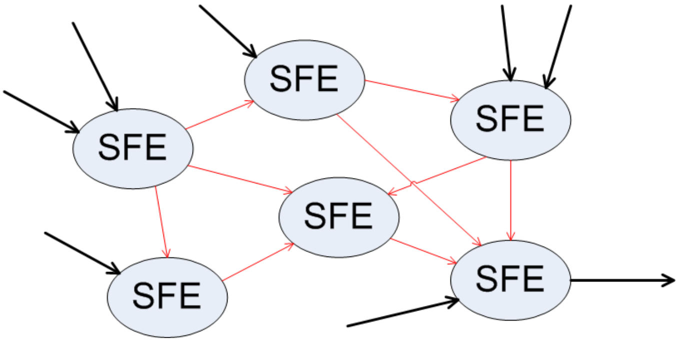
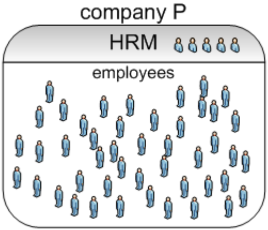


- In general, anonymous credentials with encrypted attributes can be used if the user is not allowed or does not want to know the attributes

Medical Data of Employees



- HRM (human resource management) needs to mine privacy sensitive medical data from employees

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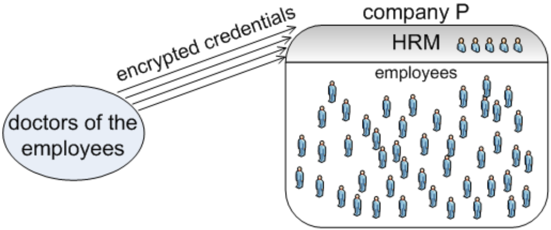

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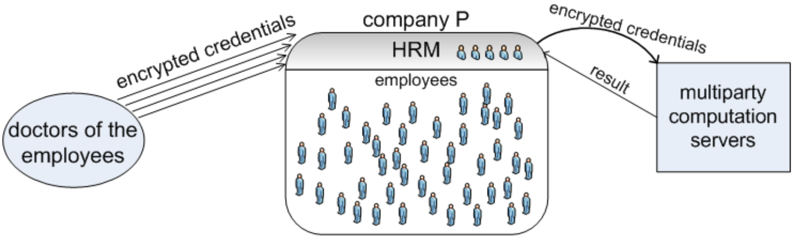


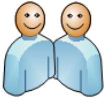
- HRM (human resource management) needs to mine privacy sensitive medical data from employees
- HRM can send these encrypted credentials to SFE servers for analysis

- ... encrypted DNA, encrypted parts of EPD (electronic patient dossier)

Boy or Girl?

- Alice and Bob want to buy/receive clothes and toys for unborn baby

Boy or Girl?



- Alice and Bob want to buy/receive clothes and toys for unborn baby
- They do not want to know the gender yet

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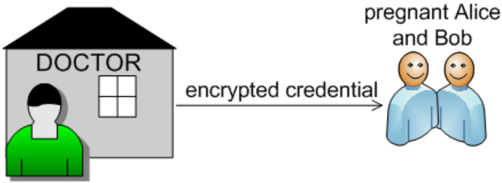

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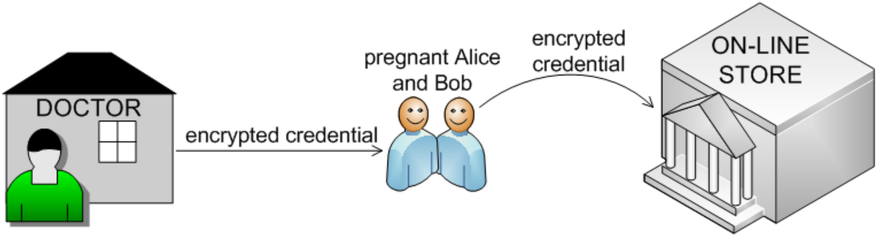

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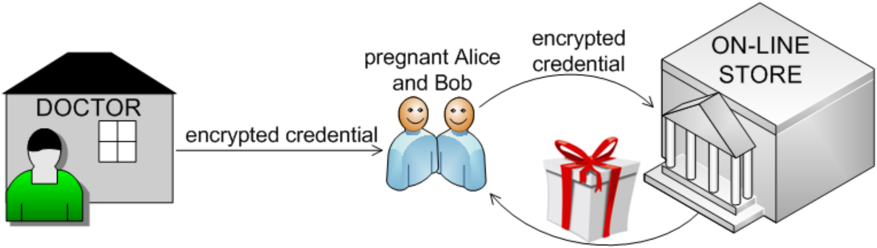

- Alice and Bob want to buy/receive clothes and toys for unborn baby
- They do not want to know the gender yet
- They unwrap the presents as soon as the baby is born!

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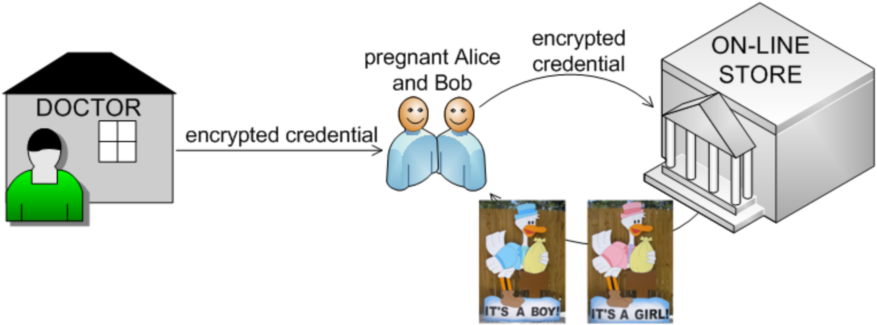

- Alice and Bob want to buy/receive clothes and toys for unborn baby
- They do not want to know the gender yet
- They unwrap the presents as soon as the baby is born!

- ... get yard signs in advance

Outline

ElGamal Cryptosystem

- We use the ElGamal cryptosystem:
  - Group $\langle g \rangle$ of prime order q
  - Secret key $\lambda \in_R \mathbb{Z}_q$, public key $f = g^\lambda$
  - Encryption of $x \in \mathbb{Z}_q$: $[\![x]\!] = (g^r, g^x f^r)$ for $r \in_R \mathbb{Z}_q$

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- Homomorphic properties:
  - Addition: $[\![x]\!][\![y]\!] = [\![x+y]\!]$
  - Multiplication by constant: $[\![x]\!]^c = [\![xc]\!]$
  - Re-randomization: $[\![x]\!][\![0]\!] = [\![x]\!]$

Outline

Anonymous Credential Schemes

- Three components:
  - Key generation: algorithm for $\mathcal{I}$
  - Issuance: protocol for $(\mathcal{I}, \mathcal{U})$
  - Verification: protocol for $(\mathcal{U}, \mathcal{V})$

- Credentials are tuples (p, s, σ), where:
  - p public part, and σ signature on p
  - s secret part corresponding to p

- s contains the attributes on which the credential is issued
- In this presentation: 2 attributes (x_1, x_2)

Brands' Anonymous Credential Scheme

- Brands' credential schemes: single-use credentials
- We use the Brands' credential scheme based on the blind Chaum-Pedersen (CP) signature scheme
  - Issuance possible without $\mathcal{I}$ learning attributes
  - This variant is also used in U-Prove

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- Key generation: group $\langle g \rangle$ of prime order q
  Secret key $x_0, y_1, y_2 \in_R \mathbb{Z}_q$, public $h_0 = g^{x_0}$, $g_1 = g^{y_1}$, $g_2 = g^{y_2}$

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- Credential on (x_1, x_2) is a tuple $(\underbrace{h'}_{p}, \underbrace{x_1, x_2, \alpha}_{s}, \sigma)$ s.t.:
  a) σ is CP-signature on h'
  b) $(g_1^{x_1} g_2^{x_2} h_0)^\alpha = h'$

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- Issuance: $\mathcal{I}$ issues blind CP-signature σ on $h = (h')^{1/\alpha}$

$\mathcal{U}$		$\mathcal{I}$
(knows: x_1, x_2, α; h, h')		(knows: x_0; h)
		$\ldots$
	$\xleftarrow{z = h^{x_0}\,;\, a, b}$	
$\ldots$		
	$\xrightarrow{c}$	
		$\ldots$
	$\xleftarrow{r}$	
$\ldots$		

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\ldots & & \\
 & \xrightarrow{c} & \\
 & & \ldots \\
 & \xleftarrow{r} & \\
\ldots & &
\end{array}
$$

- Verification: $\mathcal{U}$ proves knowledge of x_1, x_2, α s.t. $(g_1^{x_1} g_2^{x_2} h_0)^\alpha = h'$

Extension to Encrypted Attributes

- We extend Brands' credential scheme with encrypted attributes

- Several variations discussed in the paper, where
  - $\rightarrow$ none of the participants learns the attributes
  - $\rightarrow$ all parties learn a specific (possibly different) set of attributes
  - $\rightarrow$ $\mathcal{I}$ learns the attributes, but $\mathcal{U}, \mathcal{V}$ *do not* learn these

- Now: simplified version of the encrypted credential scheme

Extension to Encrypted Attributes

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- Encrypted credential on (c_1, c_2) is a tuple $(\underbrace{h', c_1', c_2'}_{p}, \underbrace{\alpha}_{s}, \sigma)$ s.t.:
  a)
  b)

- Now, encryptions $c_1 = [\![x_1]\!]$, $c_2 = [\![x_2]\!]$ belong to public part

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  b) $(D((c_1')^{y_1}(c_2')^{y_2})h_0)^\alpha = h'$

- Now, encryptions $c_1 = [\![x_1]\!]$, $c_2 = [\![x_2]\!]$ belong to public part
  $\rightarrow \mathcal{U}$ has to re-randomize c_1, c_2 in issuance
  $\rightarrow \mathcal{U}$ cannot prove knowledge of x_1, x_2 in verification

Technical Issues

$\mathcal{U}$ has to re-randomize $c_1 = [\![x_1]\!], c_2 = [\![x_2]\!]$ in issuance

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Problem: user can replace c_1 by $[\![0]\!]$ and obtain oracle for "$x_1 = 0$?"

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Solution: credential will actually be issued on $[\![x_i + \phi_i]\!]$, for $\phi_i \in_R \mathbb{Z}_q$

- g^{ϕ_i} can be made public
- $[\![x_i]\!]$ can be obtained from $[\![x_i + \phi_i]\!]$ and g^{ϕ_i}

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Verification: $\mathcal{U}$ proves knowledge of α s.t. $(D((c_1')^{y_1}(c_2')^{y_2})h_0)^{\alpha} = h'$

- Verifier needs secret data, namely y_1, y_2, and λ (for decryption)
- Verifier is required to be semi-honest (threshold cryptography)

Security Analysis

- Proven secure, against:
  - Malicious $\mathcal{I}$ and $\mathcal{U}$
  - Semi-honest $\mathcal{V}$ (threshold cryptography)

- Security based on:
  - DDH assumption
  - Random Oracle Model
  - Security of blind Chaum-Pedersen scheme
  - A new assumption
- Same level of security as original Brands' scheme

Conclusions

- We introduced anonymous credential schemes with encrypted attributes, and presented and analyzed various efficient constructions based on a credential scheme by Brands
- Wide range of applications:
  - Missing link between credential schemes and SFE
  - Medical data of employees, boy or girl?, ...
  - Letters of recommendation, medical data of illnesses, ...
- Further research:
  - Encrypted credential schemes with multi-use credentials
  - Public verifiability of encrypted credentials

Thank you for your attention!

SUPPORTING SLIDES!!!

Σ-protocols

- Simplest example: Schnorr's identification protocol
- We consider a group $\langle g \rangle$, and public $h \in \langle g \rangle$
- Prover wants to prove that he knows $x = \log_g h$

Σ-protocols

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$$
\begin{array}{rcl}
\textbf{Prover} & & \textbf{Verifier} \\
\text{(knows: } h; x\text{)} & & \text{(knows: } h\text{)} \\
u \in_R \mathbb{Z}_q, \quad a \leftarrow g^u & \xrightarrow{\quad a \quad} & \\
 & \xleftarrow{\quad c \quad} & c \in_R \mathbb{Z}_q \\
r \leftarrow u + cx \bmod q & \xrightarrow{\quad r \quad} & \\
 & & g^r \stackrel{?}{=} ah^c
\end{array}
$$

Σ-protocols

- Simplest example: Schnorr's identification protocol
- We consider a group $\langle g \rangle$, and public $h \in \langle g \rangle$
- Prover wants to prove that he knows $x = \log_g h$

Prover		**Verifier**
(knows: $h; x$)		(knows: h)
$u \in_R \mathbb{Z}_q, \quad a \leftarrow g^u$	$\xrightarrow{a}$	
	$\xleftarrow{c}$	$c \in_R \mathbb{Z}_q$
$r \leftarrow u + cx \bmod q$	$\xrightarrow{r}$	
		$g^r \stackrel{?}{=} ah^c$

- This is Σ-protocol for relation $\{(h; x) \mid h = g^x\}$
- Σ-protocol: completeness, special-soundness, honest-verifier zero-knowledge

Key Generation for $\mathcal{I}$

- **Public:** group $\langle g \rangle$ of prime order q; $h_0 \in \langle g \rangle$, $g_1, g_2 \in_R \langle g \rangle$
- **Secret:** $x_0 \in_R \mathbb{Z}_q$ such that $h_0 = g^{x_0}$

- A credential on (x_1, x_2) is a tuple $(\underbrace{h'}_{p}, \underbrace{x_1, x_2, \alpha}_{s}, \underbrace{z', c', r'}_{\sigma})$ s.t.:

$$c' = \mathcal{H}(h', z', g^{r'} h_0^{-c'}, (h')^{r'} (z')^{-c'}) \text{ and } (g_1^{x_1} g_2^{x_2} h_0)^{\alpha} = h'$$

Issuance for $(\mathcal{I}, \mathcal{U})$

- For (x_1, x_2), $\mathcal{U}$ and $\mathcal{I}$ compute $h = g_1^{x_1} g_2^{x_2} h_0$

$$
\begin{array}{rcl}
\mathcal{U} & & \mathcal{I} \\
 & & z \leftarrow h^{x_0}, \quad w \in_R \mathbb{Z}_q \\
 & \xleftarrow{\quad z;a,b \quad} & a \leftarrow g^w, \quad b \leftarrow h^w \\
\alpha \in_R \mathbb{Z}_q^*, \quad \beta, \gamma \in_R \mathbb{Z}_q & & \\
h' \leftarrow h^\alpha, \quad z' \leftarrow z^\alpha & & \\
a' \leftarrow h_0^\beta g^\gamma a & & \\
b' \leftarrow (z')^\beta (h')^\gamma b^\alpha & & \\
c' \leftarrow \mathcal{H}(h', z', a', b') & & \\
c \leftarrow c' + \beta \bmod q & \xrightarrow{\quad c \quad} & \\
 & \xleftarrow{\quad r \quad} & r \leftarrow cx_0 + w \bmod q \\
a \stackrel{?}{=} g^r h_0^{-c}, \quad b \stackrel{?}{=} h^r z^{-c} & & \\
r' \leftarrow r + \gamma \bmod q & &
\end{array}
$$

- Note: $c' = \mathcal{H}(h', z', g^{r'} h_0^{-c'}, (h')^{r'} (z')^{-c'})$ and $(g_1^{x_1} g_2^{x_2} h_0)^\alpha = h'$

Verification for $(\mathcal{U}, \mathcal{V})$

- Brands' credential is $(h', x_1, x_2, \alpha, z', c', r')$ such that:

$$c' = \mathcal{H}(h', z', g^{r'} h_0^{-c'}, (h')^{r'} (z')^{-c'}) \text{ and } (g_1^{x_1} g_2^{x_2} h_0)^{\alpha} = h'$$

$$\begin{array}{rcl}
\mathcal{U} & & \mathcal{V} \\
\text{(knows: } h', z', c', r'; x_1, x_2, \alpha) & & \\
u_1, u_2, u_\alpha \in_R \mathbb{Z}_q & & \\
a \leftarrow (h')^{u_\alpha} g_1^{-u_1} \cdots g_l^{-u_l} & \xrightarrow{a; h', z', c', r'} & \\
 & \xleftarrow{\quad c \quad} & c \in_R \mathbb{Z}_q \\
(r_i \leftarrow u_i + c x_i \bmod q)_{i=1}^2 & & \\
r_\alpha \leftarrow u_\alpha + c\alpha^{-1} \bmod q & \xrightarrow{r_1, r_2, r_\alpha} & \\
 & & c' \stackrel{?}{=} \mathcal{H}(h', z', g^{r'} h_0^{-c'}, (h')^{r'} (z')^{-c'}) \\
 & & (h')^{r_\alpha} g_1^{-r_1} \cdots g_l^{-r_l} \stackrel{?}{=} a h_0^c
\end{array}$$

- Σ-protocol for $\{(h'; x_1, x_2, \alpha) \mid h_0 = (h')^{\alpha^{-1}} g_1^{-x_1} g_2^{-x_2} \wedge \alpha \neq 0\}$

Key Generation for $(\mathcal{I}, \mathcal{V})$

- **Public:** group $\langle g \rangle$ of prime order q; $h_0, g_1, g_2, f, \hat{f}, f_1 \in \langle g \rangle$
- **Secret:** $x_0, \phi_1 \in_R \mathbb{Z}_q$ for $\mathcal{I}$ and $y_1, y_2, \lambda \in_R \mathbb{Z}_q$ for $\mathcal{V}$ such that

$$h_0 = g^{x_0} \quad g_1 = g^{y_1} \quad g_2 = g^{y_2} \quad f = g^{\lambda} \quad \hat{f} = f^{x_0} = h_0^{\lambda} \quad f_1 = g^{\phi_1}$$

- A credential on x_1^* is a tuple $(\underbrace{h', c_1', c_2'}_{p}, \underbrace{\alpha}_{s}, \underbrace{z', z_1', z_2', c', r'}_{\sigma})$, where $c_1' = [\![x_1^* + \phi_1]\!]$, such that:

$$c' = \mathcal{H}([c_i', z_i', (c_i')^{r'}(z_i')^{-c'}]_{i=1}^{2}; h', z', g^{r'}h_0^{-c'}, (h')^{r'}(z')^{-c'})$$

$$\text{and } (D((c_1')^{y_1}(c_2')^{y_2})h_0)^{\alpha} = h'$$

- Note the transformation from $[\![x_1^*]\!]$ to $[\![x_1^* + \phi_1]\!]$
  - Otherwise, a malicious $\mathcal{U}'$ can replace c_1' by $[\![0]\!]$
  - Verification succeeds *if and only if* $x_1^* = 0$
    $\rightarrow$ this gives $\mathcal{U}'$ an oracle for '$x_1^* = 0$?'

Issuance for $(\mathcal{I}, \mathcal{U})$

- Credential issuance on $x_1^* \in \{0,1\}$

$$
\begin{array}{lcl}
\mathcal{U} & & \mathcal{I} \\
 & & r_1, r_2, x_I \in_R \mathbb{Z}_q \\
 & & c_1 \leftarrow (g^{r_1}, g^{x_1^*} f_1 f^{r_1}) \\
 & & c_2 \leftarrow (g^{r_2}, g^{x_2} f^{r_2}) \\
 & & h \leftarrow g_1^{x_1^*+\phi_1} g_2^{x_2} h_0 \\
 & & z \leftarrow h^{x_0}, \quad (z_i \leftarrow c_i^{x_0})_{i=1}^2 \\
 & & w \in_R \mathbb{Z}_q, \quad a \leftarrow g^w, \quad b \leftarrow h^w \\
 & & \tilde{f} \leftarrow f^w, \quad (e_i \leftarrow c_i^w)_{i=1}^2 \\
 & \xleftarrow{\substack{h, z, (c_i, z_i)_{i=1}^2; \\ a, b, \tilde{f}, (e_i)_{i=1}^2}} & \\
\ldots & &
\end{array}
$$

- Notice that also $z_i = c_1^{x_0}$ and $e_{i0} = c_i^{w_0}$ are computed

$\rightarrow$

Issuance for $(\mathcal{I}, \mathcal{U})$

- $\mathcal{U}$ and $\mathcal{I}$ know $h, z, c_1, c_2, z_1, z_2, a_0, b_0, \tilde{f}, (e_{i0})_{i=1}^2$

$$\alpha \in_R \mathbb{Z}_q^*, \quad \beta, \gamma \in_R \mathbb{Z}_q$$

$$h' \leftarrow h^\alpha, \quad z' \leftarrow z^\alpha$$

$$a' \leftarrow h_0^\beta g^\gamma a, \quad b' \leftarrow (z')^\beta (h')^\gamma b^\alpha$$

$$\left(\begin{array}{r} \delta_i \in_R \mathbb{Z}_q, \quad c_i' \leftarrow c_i \cdot (g, f)^{\delta_i} \\ z_i' \leftarrow z_i \cdot (h_0, \hat{f})^{\delta_i} \\ e_i' \leftarrow (z_i')^\beta (c_i')^\gamma e_i \cdot (a, \tilde{f})^{\delta_i} \end{array} \right)_{i=1}^2$$

$$c' \leftarrow \mathcal{H}([c_i', z_i', e_i']_{i=1}^2; h', z', a', b')$$

$$c \leftarrow c' + \beta \bmod q \quad \xrightarrow{\ c\ }$$

$$\xleftarrow{\ r\ } \quad r \leftarrow cx_0 + w \bmod q$$

$$a \stackrel{?}{=} g^r h_0^{-c}, \quad b \stackrel{?}{=} h^r z^{-c}$$

$$\tilde{f} \stackrel{?}{=} f^r \hat{f}^{-c}, \quad (e_i \stackrel{?}{=} c_i^r z_i^{-c})_{i=1}^2$$

$$r' \leftarrow r + \gamma \bmod q$$

Verification for $(\mathcal{U}, \mathcal{V})$

- $\mathcal{U}$ proves knowledge of α such that $(D((c_1')^{y_1}(c_2')^{y_2})h_0)^{\alpha} = h'$

$$
\begin{array}{rcl}
\mathcal{U} & & \mathcal{V} \\
\text{(knows: } h', c_1', c_2'; \alpha) & & \text{(knows: } h', c_1', c_2'; y_1, y_2, \lambda) \\
u \in_R \mathbb{Z}_q, \quad a \leftarrow (h')^u & \xrightarrow{a} & \\
 & \xleftarrow{c} & c \in_R \mathbb{Z}_q \\
r \leftarrow u + c\alpha^{-1} \bmod q & \xrightarrow{r} & \\
 & & b \leftarrow \left[(h')^r \stackrel{?}{=} a(D((c_1')^{y_1}(c_2')^{y_2})h_0)^c\right] \\
 & \xleftarrow{b} &
\end{array}
$$

- Protocol is a zero-knowledge proof of knowledge for α such that $(D((c_1')^{y_1}(c_2')^{y_2})h_0)^{\alpha} = h'$
- $\mathcal{V}$ is required to be semi-honest (threshold cryptography)

Verification for $(\mathcal{U}, \mathcal{V})$

- $\mathcal{U}$ proves knowledge of α such that $(D((c_1')^{y_1}(c_2')^{y_2})h_0)^{\alpha} = h'$

$$
\begin{array}{ccc}
\mathcal{U} & & \mathcal{V} \\
(\text{knows: } h', c_1', c_2'; \alpha) & & (\text{knows: } h', c_1', c_2'; y_1, y_2, \lambda) \\
u \in_R \mathbb{Z}_q, \quad a \leftarrow (h')^u & \xrightarrow{\quad a \quad} & \\
 & \xleftarrow{\quad c \quad} & c \in_R \mathbb{Z}_q \\
r \leftarrow u + c\alpha^{-1} \bmod q & \xrightarrow{\quad r \quad} & \\
 & \xleftarrow{\quad b \quad} & b \leftarrow \left[(h')^r \stackrel{?}{=} a(D((c_1')^{y_1}(c_2')^{y_2})h_0)^c\right]
\end{array}
$$

- Protocol is a zero-knowledge proof of knowledge for α such that $(D((c_1')^{y_1}(c_2')^{y_2})h_0)^{\alpha} = h'$
- $\mathcal{V}$ is required to be semi-honest (threshold cryptography)
- $\mathcal{V}$ can obtain $[\![x_1^*]\!]$ by computing $c_1' \cdot (1, f_1^{-1})$:

$$c_1' \cdot (1, f_1^{-1}) = (g^r, g^{x_1^* + \phi_1} f^r) \cdot (1, g^{-\phi_1}) = (g^r, g^{x_1^*} f^r) = [\![x_1^*]\!]$$

Assumption

- $\mathcal{U}'$ is issued K credentials on x_j^* $(j = 1, \ldots, K)$, and learned $(c_{ji})_{i=1}^2$
- Then he outputs a tuple $(h', c_1', c_2, \alpha, z', z_1', z_2', c', r')$. Now, either
  - This tuple is not a valid credential
  - There exists a j such that

$\mathcal{U}'$ knows values β_1, β_2 such that $(c_i')_{i=1}^2 = (c_{ji}(g, f)^{\beta_i})_{i=1}^2$

One-more Unforgeability

- ‘Hard to obtain $K+1$ credentials after K issuing executions’
- Credential scheme is based on blind Chaum-Pedersen signature scheme
- Reducing one-more forgeries:

$$\begin{array}{ccccc} \mathcal{U}' & & \mathcal{F} & & \mathcal{S}_{CP} \\ \left[\right. & \xleftarrow{\text{issues credential}} & & \xleftarrow{\text{issues CP-signature}} & \left.\right]\ K \text{ times} \\ & \xrightarrow{\text{forgery: } K+1 \text{ credentials}} & & \xrightarrow{\text{forgery: } K+1 \text{ CP-signatures}} & \end{array}$$

- Our scheme is at least as secure against one-more forgeries

Verification Protocol

- Recall the verification protocol

$$
\begin{array}{rcl}
\mathcal{U} & & \mathcal{V} \\
(\text{knows: } h', c_1', c_2'; \alpha) & & (\text{knows: } h', c_1', c_2'; y_1, y_2, \lambda) \\
u \in_R \mathbb{Z}_q, \quad a \leftarrow (h')^u & \xrightarrow{\quad a \quad} & \\
 & \xleftarrow{\quad c \quad} & c \in_R \mathbb{Z}_q \\
r \leftarrow u + c\alpha^{-1} \bmod q & \xrightarrow{\quad r \quad} & \\
 & \xleftarrow{\quad b \quad} & b \leftarrow \left[(h')^r \stackrel{?}{=} a(D((c_1')^{y_1}(c_2')^{y_2})h_0)^c \right]
\end{array}
$$

- Protocol should be a secure proof of knowledge
  - Proof of knowledge: *complete* and *special sound*
  - Secure: views on protocol *simulateable* for passive $\mathcal{V}'$ and active $\mathcal{U}'$
- Simulation of view of active $\mathcal{U}'$: after sending r, $\mathcal{U}'$ already knows b